

Under the Influence of Malthus's Law of Population Growth: Darwin Eschews The Statistical Techniques
of Aldolphe Quetelet

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It must seem strange in view of Quetelet's early advocacy of Statistics as an educational discipline, that so many leading, and by the standards of their time, well educated, men were quite unaware that they had anything to learn in this field.

R.A. Fisher, "The Expansion of Statistics", 1953.

O. INTRODUCTION

In the epigraph, Fisher is blaming two generations of theoretical biologists, from Darwin on, for ignoring Quetelet's statistical techniques and hence harboring confusions about evolution and natural selection. He is right to imply that Darwin and his contemporaries were aware of the core of Quetelet's work. Quetelet's seminal monograph, *Sur L'homme*, was widely discussed in Darwin's academic circles. We know that Darwin owned a copy (Schweber 1977). More importantly, we have in Darwin's notebooks two entries referring to Quetelet's work on the cause of a large-scale global phenomenon where each year more boys were born than girls. The first entry is written sometime between April and July 1838. Darwin writes "Find out from the Statistical Society—where M. Quetelet has published his laws about sexes relative to age of Marriages" (C 268, Barrett *et al.*, 1987, p. 324). The second is written sometime after October 16, 1838: "In the Atheneum Numbers 406, 407, 409, Quetelet papers are given, & I think facts there mentioned about proportion of sexes, at birth & causes" (*Ibid*, p. 379). So, even if Darwin did not read *Sur L'homme* directly it is likely (though not certain) that he read its review in the *Atheneum*. There is no doubt that Darwin eventually became familiar with Quetelet's work in statistics. The smoking gun is an essay that Darwin writes in 1874, entitled, "On the Males and Complemental Males of Certain Cirripedes, and on Rudimentary Structures" where he discusses Quetelet's laws of variation.

As for Darwin's contemporaries Fisher is wrong to imply they were not aware of the importance of Quetelet's work. There is little question about his impact on James Clerk Maxwell and on Darwin's cousin, Francis Galton. Both read the extensive review of Quetelet's achievements in statistics in John Herschel's 1850 essay in *Edinburgh Review* (then "anonymous"). Both were profoundly affected by it (Porter 1986, Gillespie 1961, Brush 1965, Hacking 1990). Maxwell, following Quetelet *via* Herschel, takes his basic assumption that the aggregate description of the numerous collisions between individual molecules in a gas fit the regular distribution patterns of gas laws already known at the time (Brush 1965, p. 186). Galton's central postulate in *Heredity Genius* that drives his theory of inheritance is derived explicitly from deliberating upon Quetelet's use of Gaussian bell-shaped curves (Hacking 1990, 184 who provides quotes from Galton on his reflections on Quetelet).

If Darwin, Galton, and Maxwell were all exposed to Quetelet's early advocacy of statistics and Galton and Maxwell were aware that they had something to learn in this field, why was Darwin apparently unaware? This is the question that drives this essay.

Darwin's sons, Frank and Leonard, reported to Karl Pearson that their father had a "non-statistical" mind. This came in response to Pearson's inquiry about whether their father was aware that the theory of natural selection is applicable to statistical analysis (Porter 1986, p. 135). This may be part of the answer,

but, as Stephen Jay Gould used to say, attempts to disprove Darwin's genius usually end in failure. Rather, I suggest that we inquire as to what Darwin was thinking about such that Quetelet's statistical techniques would have provided him an answer. We will find that by the time Darwin inquired into Quetelet he had already all he needed in Malthus's law of population growth. Let me put the point a different way. Suppose Darwin *was* a great statistical thinker. Even so, Darwin would have found no reason to abandon Malthus's law of population growth. Part of the reason depends on what Darwin was thinking about at the time we was reading Malthus. Part of the reason concerns misinformation Darwin might have received from the source of his knowledge of Quetelet. I discuss both reasons in this essay.

1. THE POPULATION PHENOMENON

In general Darwin, Maxwell, and Galton were all looking for ways to analyze various instances of what I will call "population phenomena", large-scale regularities that belie individual differences; or, as Porter puts it, regularities that appear to be true at the level of populations but not necessarily true for any particular individual (Porter 1986). Examples of population phenomena come largely from demography, including stable death rates, birth rates, population growth rates, and crime rates for a population over long periods of time. The puzzle about these stable rates was well expressed by André Guerry as he reflected on the emergence of stable crime rates out of multiple causes. He asked, if we consider the infinite number of circumstances that lead to a crime, how can we fathom that their conjunction reveals a constant effect? ("Essai sur la statistique morale de la France", 1883, p. 11, quoted in Porter, 1986, p. 49). The skewed sex ratio is a particularly old example, dating back to John Graunt's discovery in 1610 that christening records in London over the years demonstrated that more boys are born than girls (Sober ms.). It would be difficult to fathom from the various ratios of boys to girls between households that a large-scale skew towards the birth of boys emerges when a large number of families are censused.

Quetelet's contribution to demography was to show that a large-scale distribution pattern, such as a stable average or a bell-shaped Gaussian curve, is what we would expect to see if individuals in a very large population share some common causal features even if these features weren't experienced by all individuals or weren't experienced all in the same way. He formulated his "fundamental principle" for a new "social physics" according to this statistical insight: "*the greater the number of individuals observed, the more do individual peculiarities, whether physical or moral, become effaced, and leave in a prominent point of view the general facts, by virtue of which society exists and is preserved*" (1842, p. 6, his italics). Quetelet was best known for his (excessive) rhetoric about the social importance of the "average man". However, as Herschel, Maxwell, Galton, and Fisher, among others, recognized, Quetelet's techniques on analyzing common causal features of statistical distributions was much more important.¹

¹My exclusion of the Anonymous *Athenium* reviewer is intentional. Hilts (1973) among others commentators, pay insufficient attention to this pioneering aspect of Quetelet's work. Like Anonymous, they are too distracted by Quetelet's excessive rhetoric. For distinguishing rhetoric from technique, Herschel (1850) and Stigler (1986) are commendable.

It is fairly easy to demonstrate that Maxwell and Galton were interested in some manifestation of the population phenomenon. Maxwell found it in gases where the laws of diffusion belie the variegated motion of gas molecules. Galton found it in human inheritance. For some human characters, height and "intelligence", for example, parents tend to resemble their offspring in the short run of few generations. Yet, in the long run, the lineage tend to revert to "mediocrity".

Darwin was interested in several instances of the population phenomenon, including extinction, speciation and adaptation. The puzzle for adaptive speciation is how is it possible that out of individual differences evolves clustering of species that are so well adapted to their local environmental conditions? The historical question is, which of these (if any) was he interested in when he first queried about Quetelet's work? There is a bit of confusion among Darwin biographers on this question. Let me tell two stories, one is convention but false about chronology in regard to Darwin's interest in Quetelet. The second is an addendum, getting the chronology right.

The conventional story comes mostly from Schweber (1977) and Gruber (1974). Darwin's main motivation was to provide an alternative to Paley's divine interventionist's explanation for the exquisiteness fit of natural forms to local environmental conditions. For Paley, species are immutable, their form is imposed by God. For Darwin, species evolve to adapt to the environment. It is in the description of evolution that Darwin generates his problem of variation akin to that in the social sciences which prompts Darwin to turn to Malthus and Quetelet.

On this view, Darwin's phenomenon of variation emerges from the contrast between species that are well adapted and the morphological differences among individuals within the species. That individuals within a species vary is true by simple observation. Darwin's own notebook entries that lead up to the development of his theory of natural selection are full of examples of how impressed he is with individual variation. For example, Darwin writes back in September 23, 1838, "Saw in Loddiges garden 1279 varieties of roses!!! Proof of capability of variation" (D118, in Notebooks, p. 371). Gruber (1974) argues that passages like this one demonstrate that Darwin distinguished between the cause of variation and the cause of adaptation as early as the end of 1838.

Darwin further hypothesized that, contrary to the Lamarckian view, the cause of individual differences is independent of the cause of adaptation. Darwin attributed the cause of variation to "chance" (meaning that the cause is indeterminate). Variation is the starting point, a condition for evolution by natural selection. Darwin had no good idea how individuals come to differ from one another. So, the problem of variation for Darwin would be how could adaptations arise by the chaos of individual differences? Darwin's inquiry into Quetelet's analysis of sex ratio would have taken him to the heart of Quetelet's analysis of variance perhaps providing him a clue as to how to analyze the emergence of stable regularities out of the chaos of individual differences without having to resort to divine intervention. Instead, Darwin sees in the *Atheneum* review, an explicit endorsement of Malthus's law of population growth given Darwin the instigation needed to complete his theory of natural selection. The upshot is that

the *Athenaeum* review leads Darwin straight to Malthus and hence, straight to his version of natural selection (Schweber 1977).

Yet, by Spring or Summer of 1838, when Darwin expressed his specific interest in Quetelet's work, he had not yet conceived of the independence of individual variation and adaptation. By then, Darwin still thought that the cause of variation and adaptation were the same. According to Hodge and Kohn (1985) the rupture of variation and adaptation was not constructed until at least late November or early 1838. Further, according to Schweber 1977, Darwin's discovery of the *Athenaeum* review dates September 11, 1838, two weeks before Darwin reads Malthus on populations and expresses his first formulation of natural selection on the origin of adaptation. But, Kohn (in Barrett *et. al.*, 1987, p. 375) points out that the ink used in the notice about the *Athenaeum* review is brown, distinct from the grey ink used predominantly in the D notebooks. The shade of brown matches that of early entries in the E notebooks that date to at least October 16, 1838. So, Schweber's theory that the review of Quetelet is the catalyst for Darwin's returning to Malthus cannot be correct. Of course, it is possible that Darwin revisited Quetelet shortly after December 1838. After all, Darwin discusses Quetelet's work on variation in 1874. More plausibly, however, Darwin did not think he needed Quetelet's methods to analyze how stable adaptive clusters emerge from the chaos of individual variation. For, in the meantime, Darwin reads Malthus on population growth (around September 28, 1838) and eventually becomes convinced, over a period of many months, that Malthus's law is sufficient to explain any such population phenomena.

The questions to ask at this point are what was Darwin looking for when he encountered Malthus? What did Darwin find in Malthus that would have him thought Quetelet's statistical methodology unnecessary? We need to turn to the notebooks to answer these questions.

2.1. DARWIN ON EXTINCTION

The answer is that Darwin was interested in how extinctions, a population phenomenon, emerge out of a variety of causes. Specifically, in the following passage written late in 1837, Darwin wondered how a species might go extinct without the appearance of a *single* cause.

"In looking at two fine families one with B successors <for> centuries, the other will become extinct.—Who can analyze causes, dislike to marriage, hereditary disease, effects of contagions & accidents: yet some causes are evident, as for instance one man killing another.—So is it with *varying* races of man: these races may be overlooked mere variations consequent on climate &c—the whole races act towards each other, and are acted on, just like the two fine families <<no doubt a different set of causes must act in the two case,>> May this not be extended to all animals first consider species of cats.--<& other tribes>...." (B 147).

The two 'fine families' both experience the tragedies of life, death, aversion to marriage, disease, accidents, but one lineage propagates and the other goes extinct. Darwin asserts that the fate of the 'two fine families' problem is analogous to understanding species extinction.

Sprinkled throughout his notebooks Darwin considered factors that could lead to extinctions including de Candolle's idea of a war between organic beings and Lyell's view that rather rapid species changes could result when one species invades another (see Hodge and Kohn 1985, p. 194). Lyell's idea is roughly, that in an area already fully stocked with individuals, small changes in ecological conditions can lead to rather dramatic effects such as some species driving out others to the point of extinction. It isn't until Darwin encounters Malthus that he finds a way to comport Lyell's view of extinction with a theory of how extinction could emerge from multiple causes.

To understand why Malthus would have provided an answer to the question about extinction we should ask why sort of answer Darwin would have found acceptable. While Darwin sought to undermine divine interventionist explanations Darwin was not an atheist, *tout court*. In the early years before his essay of 1844, Darwin, following Paley, believed in the perfection of adaptation and harmony of ecosystems (Ospovat 1980). And, following Whewell and Herschel, Darwin believed God's perfections were mediated through a combination of *laws* (Brooke 2003, p. 197). Laws, here, were understood in the model of Newton's laws of motion and theory of universal gravity; they were universal and deterministic (Hull 1989). According to Whewell, the existence of Newtonian laws *necessitated* the existence of God. The perfection and harmony of laws were the hallmark of a benevolent God. Newton held the same view (see Ariew 2003). Confirming references to final causes and laws litter the early, pre-Malthus notebooks. The following is a good example where Darwin rejects interventionistic explanations for laws without denying that God is ultimately responsible for them.

September or October 1837 (B 101): "Astronomers might formely have said that God ordered, each planet to move in its particular destiny.—In same manner God orders each animal created with certain form in certain country, but how much more simple, & sublime power let attraction act according to certain law such are inevitable consequences let animal be created, then by the fixed laws of generation, such will be their successor".

Hodge, among other historians, has argued that throughout Darwin's writings he sought to conform to the *vera causa* idea. That is, the search for causes confirmable from direct independent evidence, or from facts other than those it is to explain. Again, Newton is the paradigm. Gravitation is known to exist from our direct experience of objects such as pendulum swings and falling stones (Hodge 1987, p. 236-7).

In Malthus, Darwin found what he was looking, a non-interventionist explanation of a population level phenomenon that ascribed laws in the spirit of Newtonian dynamics. Darwin read the following passage in Malthus sometime after September 28, 1838:

"In New Jersey the proportion of births to deaths, on an average of 7 years, ending with 1743, was 300 to 100. In France and England the average proportion cannot be reckoned at more than 120 to 100. Great and astonishing as this difference is, we ought not to be so wonder-struck at it, as to attribute it to the miraculous interposition of Heaven. The causes of it are not remote, latent and mysterious, but near us, round about us, and open to the investigation of every inquiring mind". (Cited in p. 397 in Notebooks): Malthus 1826, 1:529.

Darwin was impressed that Malthus explains stable death rates in terms of fixed natural laws rather than in terms of an external Godly force. With obvious elation, Darwin quotes and comments on the passage that directly follows the one I just cited:

Epidemics—seem intimately related to famines., yet very inexplicable.—do p. 529. "It accords with the most *liberal!* spirit of philosophy to believe that no stone can fall, or plant rise, without the immediate agency of the deity. But we know from *experience!* That these operations of what we call nature, have been conducted *almost!* Invariably according to fixed laws: And since the world began, the causes of population & depopulation have been probably as constant as any of the laws of nature with which we are acquainted."—this applies to one species—I would apply it not only to population & depopulation, but extermination & production of new forms.—the number & correlations.

Why the elation? No need to postulate divine intervention in an explanation of population growth, extinction, and the creation of new forms. All these phenomena exist according to fixed laws, just like stones falling and plants rising. Very interesting indeed: Malthus provides a way to explain population level phenomena in terms of fixed laws like those that govern all physical and biological processes. No need to postulate divine intervention in the explanation of population growth, extinction, and the creation of new forms.

Consistent with the "Epidemics" quote, Darwin's famous Notebook D entries (134-5e) invokes Malthus's population law to solve the problem of extinction. Darwin reads Malthus as a vindication of Lyell's theory of extinctions. Let us take a look at Darwin's vindication one step at a time.

First, Darwin reads the crush of population resulting in the positive checks of famine and death as resulting in a *war* between species over resources:

Even the energetic language of Malthus, Decandoelle, does not convey the warring of the species as inference from Malthus.—increase of brutes, must be prevented solely by positive checks, excepting that famine may stop desire.—in Nature production does not increase, whilst no checks prevail, but the positive check of famine & consequently death.

Why so dramatic as "war"? Because, as Malthus has it, the tendency of organisms to increase geometrically means that populations grow very fast. That is important for Darwin's vindication of Lyell. The all-caps in the following entry are Darwin's own: "population in increase at geometrical ratio in FAR SHORTER time than 25 years—yet until the one sentence of Malthus no one clearly perceived the great check among men". Notice, here Darwin treats the geometrical rate of growth of populations as a force *independent* of the factors make up the checks. (This will become important later on when we analyze Quetelet's critique of Malthus.) If every lineage experiences the same tendency for rapid increase and the resources are limited, Darwin reasons, this will cause a tremendous pressure on all species against each other in competition for limited resources. The resultant effect is a "warring of the species". The war is so intense because the crush of population is so great. As a result any slight change to ecological conditions may give one species an advantage over another and drive the other one out. Darwin writes: "One may say there is a force like a hundred thousand wedges trying force into every kind of adapted structure into the gaps of in the oeconomy of Nature, or rather forming gaps by thrusting out weaker ones".

I read the rest of the passage as an answer to Darwin's query, in B147, about how we can understand extinctions arising without the existence of a single cause (like famine) and without invoking an intervening God. The key is that the Malthusian "checks" themselves describe conditions of a *population* not of any particular individual. Darwin sees clearly that there are multiple causes for each check. He writes: "take Europe on an average, every species must have same number killed, year with year, by hawks. by. cold &c even one species of hawk decreasing in number must effect instantaneously all the rest". While every population experiences the same average death rate, the conditions of death might differ between lineages.

Let us put together Darwin's "two fine families" of B147 with these hawks to get an insight to Darwin's solution to the problem of how extinctions might arise by multiple causes. In general, both experience the force of the check against their inherent tendencies to geometrically increase their numbers in the same way, i.e. both are affected by the same average death rate. Yet, each experiences different causes of death. So, suppose one family's growth is more likely to be threatened by the cold climate and the other is more likely to be threatened by predator hawks, while both are limited by the same resource constraints. If the number of hawks decreases while the threat of cold remains the same, the hawk-threatened family will likely to increase their numbers at the expense of the cold-threatened family. I say "at the expense of" because, you might recall, the increase of numbers for one family takes away potential resources from the other.

Recall in the "Epidemics" quote that Darwin promises to extend the Malthusian law to explain the production of new forms. To illustrate how, consider how the two fine families differ from one another in their abilities to utilize resources and stave-off potentially fatal threats. So, given a change in conditions, say, the decrease of hawks, one family propagates its kind relatively quicker than the other. The result is a replacement of one kind (defined in terms of adapted abilities) for another in the overall population. This seems to be what Darwin had in mind when he wrote: "the final cause of all this wedging, must be to sort

out proper structure & adapt it to change". Again, consider this in the context of Darwin vindicating Lyell's view that nature is all full up. Not only is nature full but it is packed with various forms each trying to push the boundaries even further, creating a great pressure. The balance in the war between forms is tenuous, such that any small change in condition that favors one form will rapidly lead to the shoving out of one form by another (Hodge 2003).

The passage of September 28, 1838 *predates* Darwin's theory of natural selection, but the wedges and force analogy is used by Darwin in several later essays, including the essay of 1842, in the *Origins*, and in an essay entitled "Natural Selection" (see Schweber 1977, p. 297). I say "predates" because, if you recall in my criticism of the "conventional story", the Darwin we know from his initial musings about Malthus and extinctions is not the Darwin we know from the *Origins*. By 1840, Darwin decouples the cause of variation from the cause of adaptation. This move affects his teleology. He no longer sees variations as emerging towards perfection—as he did in September, 1838—whereby once perfection is achieved, selection halts. Now, the mechanism of variation (of which Darwin knows not what) produces a constant supply of variants. Natural selection is potentially in continuous operation since: a) variation is the fuel for natural selection and b) the supply of fuel is potentially inexhaustible, and c) the Malthusian conditions for a struggle are constant as the tendency of population increase is always up against the multitude of different ways nature checks the population (Ospovat 1980, p. 85). If natural selection is continuous, never stopping, the idea of perfect adaptation has to give way to a view that there is always room for improvement.

How does this affect the role of Malthus in Darwin's thought from September 28, 1838 until the publication date of the *Origins*? As many commentators point out (Hodge and Kohn 1985, Hodge (2003, p. 60, Grene and Depew 2004, p. 198), applying Malthus's laws of human population growth to non-human populations lead to Darwin eventually introducing an elaborate analogy to artificial selection, the *vera causa* of natural selection in the *Origins*. Yet, even in the *Origins*, the Malthusian law of population growth serves as the "trigger" for natural selection *via* the struggle that follows the crush of populations (Grene and Depew 2004, p. 198). As Darwin writes (notice the use of the term "force"):

A struggle for existence inevitably follows from the high rate at which all organic beings tend to increase. Every being, which during its natural lifetime produces several eggs or seeds, must suffer destruction during some period of its life, and during some season or occasional year, otherwise, on the principle of geometrical increase, its numbers would quickly become so inordinately great that no country could support the product. Hence, as more individuals are produced than can possibly survive, there must in every case be a struggle for existence, either one individual with another of the same species, or with the individuals of distinct species, or with the physical conditions of life. It is the doctrine of Malthus applied *with manifold force* to the whole animal and vegetable kingdoms; for in this case there can be no artificial increase of food, and no prudential restraint from marriage. Although some species may be now increasing,

more or less rapidly, in numbers, all cannot do so, for the world would not hold them. (*Origins*, chapter three, my italics).

So, Darwin found in Malthus a quantitative law that instigates a force that culminates in speciation, extinction, and ultimately, adaptation. Darwin in the *Origins*, it seems, does not recognize any need for Quetelet.

But then, why did Darwin inquire about Quetelet's analysis of sex ratios back in 1838? Recall that the existence of skewed sex ratios in favor of boys is, like the problem of extinction, another instance of the phenomenon of variation. The twist in the story is had Darwin looked through *Sur L'homme* he might have encountered Quetelet's powerful critique of Malthus's law of population growth law. More importantly, underlying Quetelet's criticism of Malthus is a contrast between two distinct ways to understand how growth laws from local chaos.

2. QUETELET'S CRITIQUE OF MALTHUS

Malthus's doctrine of excessive reproduction goes like this: human populations will increase by a geometric ratio if there is no check to its growth. Yet, under the most favorable circumstances for production the means of subsistence can never increase quicker than in an arithmetic ratio. The differences in the two ratios suggest that the growth of populations cannot continue forever, at some point the number of people will hit the limit of the region's resource capacity. There must be, therefore, "checks" to the population to prevent it from exceeding its limits. Malthus describes two sorts of checks, one acts by preventing the growth of a population, the other acts by repressing an increase as it begins. In analyzing the conditions that make up the "preventive" and "positive" checks, Malthus infamously concludes, "all checks to the population may be resolved by misery and vice".

Quetelet utilizes population censuses for countries across Europe to demonstrate, contrary to Malthus's declaration of 'misery, hardship and vice' that: a) densely populated countries are just as likely to incorporate the virtues of productivity and industriousness than misery and vice, and b) misery and hardship are not unique conditions of populations at their growth limit. Quetelet concludes as does Yule (1928) and Fisher (1953) after him, "...the dangers attending society have perhaps been exaggerated" (1842, p. 49).

Quetelet goes further, diagnosing the problem with the law of population growth. Accordingly, it lies not with the truth of the geometric ratio Malthus ascribes to populations growing unchecked by resource constraint. Nor is Malthus's claim that sustaining resources can grow only as an arithmetic ratio necessarily false (though Quetelet expresses some doubt in a footnote). The problem is Malthus's description of the "modus operandi" upon which the law of resource growth acts as a limit to population growth. Quetelet finds no evidence that population approaches the limit like a force hitting the impenetrable boundary of resource constraints with a "violent shock", in turn causing the conditions of misery and hardship. Rather, populations approach their limit as a stable equilibrium. (Fisher concurs

(1953, p. 5), as does Yule (1928, p. 41)). Quetelet concludes his empirical argument, "the experience of our old Europe proves very fully, that population arrives at its state of equilibrium, increases, or recedes, by generally following one law of possibility" (p. 49).

On the surface, the take home message is that Malthus's depiction of checks resolving in terms of misery and vice does not fit with empirical evidence. Yet, Quetelet's critique goes deeper. To see the point, recall that the growth law, is, like stable birth, death, skewed sex ratios, an instance of the population phenomenon. The issue concerns the relation between the large-scale regularities and the local environmental conditions from whence they (somehow) emerge. For Malthus population growth is a fixed law of nature that affects all individuals. It is as if humans are tugged by two forces, the force of their own free will and that of some higher order law which is imposed by God (but not as an intervention). Quetelet's approach is distinct.

Quetelet along with LaPlace introduced statistical techniques from astronomy to demonstrate how the large-scale regularities might be seen as emerging from the collective activity of individuals. In astronomy, the transit of a planet is not determined by extrapolating from a few observations. Observational reports are variegated and many are fraught with errors. But, if the plot of numerous observations conforms to a Gaussian bell-shaped curve, we can extrapolate from the mean, the highest point on the curve, the planet's most likely location at a particular time. Statistical techniques then provide the proper way to smooth away individual differences to detect common (frequent) causes between individuals (Quetelet, p. 5). For instance, the appearance of a Gaussian distribution curve in the data indicates the existence of numerous "constant causes" perturbed by many accidental causes.

By now we should have an impression of the difference between Malthus and Quetelet on how to understand the relation between large-scale regularities and local chaos. For Malthus, as it is (in general) for Darwin, the population level effects are fixed laws imposed by God. So, to understand population trends, individual differences can be ignored. For Quetelet and Laplace there is no higher order law to explain large-scale trends. Rather they emerge out of the aggregate of the individual level events. Statistical techniques allow us the proper perspective to view these trends (Quetelet 1842, p. ??) but it does not follow that the individuals ought always be treated as a collective. In particular the resolution between population growth and their checks ought not ignore the interactions at the individual level. This is at the core of Quetelet's diagnosis of Malthus's exaggeration: in effect, Quetelet accuses Malthus of having committed a category error, treating the resolution of the two laws as if population growth is an independent force colliding with an impenetrable boundary of resource constraint. Rather, Quetelet thinks that the resolution depends on interaction between the numerous individual level causes. Hence, the individual level factors that determine population growth are not independent of the factors that determine resource growth. The following quote exposes the contrast in the two views:

"[Contrary to Malthus] population can never be developed so rapidly as to strike suddenly against this bound the obstacles which previously arise, having the same tendency, are too numerous to render a violent

shock possible. Nature does not raise a smaller tribute of death; but since we pay this tribute in detail, it is less sensible to us than if we required suddenly to discharge it" (p. 49, my italics).

The 'detail' here refers to the individual causal events that make up the large-scale regularity. Quetelet's 'resolution by the interaction of innumerable causes' is again evident in his analogy between populations oscillating around a stable equilibrium and a cloud ascending and descending in the sky:

"In the midst of the causes innumerable which may disturb this state of equilibrium, population advances or recedes almost in the same manner as we see the cloud ascend or descend according to the temperature, direction of the winds, and a crowd of other atmospherical circumstances, which, however, does not prevent its always reaching a certain average height, depending on its constitution and the obstacle which the resistance of the air opposes to its descent" (p. 49).

As with clouds, Quetelet believes that in all human cases the growth of populations approaches its limit as a stable equilibrium. But the intensity of the interaction of the numerous causes that determine how populations approach their stable growth limit is contingent upon the particular circumstances facing a population. To study this, Quetelet urges case-by-case statistical analysis for each of the populations under study:

When the social system undergoes any changes, the obstacles always preserve the same mode of action [equilibrium]; but their intensity may vary in an infinite manner, so that the development of population may be infinitely modified likewise. If we possessed exact census for different periods, the analysis would show the intensity of the causes which have been able to accelerate or oppose the development of the population, and the circumstances which have given origin to them (p. 49).

Quetelet warns against using short-term data in support of determining the state of a country. "Many authors only estimate the increase of population from one or two years of observation, and are thus exposed to the greatest errors." The error in this case is "mixing the influences which we wish to determine, with those resulting from an infinity of causes, which may often cause the former to be entirely misunderstood". I read Quetelet as warning against predetermining the causes that the data is supposed to reveal. I cannot help but think that Quetelet has Malthus in mind when he writes these warnings.

If Quetelet's diagnosis of Malthus's thinking is correct, we can see how Malthus might have thought that 'misery and vice' follows the crush of population growth against the limits of resources. Yet, if Quetelet's own view is correct, the resolution of the growth laws is determined by the numerous and contingent events that take place at the individual level. Consequently, on Quetelet's (Yule's and Fisher's) view, there is no need for the imposition of misery and vice to the conditions of populations at their stable

growth limit.²

Let us assume that Darwin did not read these passages in *Sur l'homme*. Might have he read about them in the anonymous review of Quetelet in the *Atheneum*? Specifically, what does Anonymous say about Quetelet's critique of Malthus?

3. REVIEW OF QUETELET'S *SUR L'HOMME* IN THE *ATHENEUM*.

Anonymous mentions Malthus or the consequence of the crush of populations in two places. In neither case Anonymous is sensitive to the deeper message of Quetelet's critique of Malthus's law of population growth. In fact, at least in the first instance, Anonymous, ironically, reinforces the value of Malthus's law. First, just after Anonymous's discussion of Quetelet's work on the cause of sex ratios, he or she cites Quetelet's view that "there exists a fixed relation between mortality and fecundity, or that the number of births is regulated by that of the deaths". Anonymous responds to the statement in Malthusian fashion:

In a certain sense this is true; for, supposing an epidemic to have thinned a population, it is to be presumed that the next generation will marry earlier and in greater numbers: but, as a general proposition, it should seem that the deaths are rather to be considered as a dependency on the births, than as a cause of their increase. *On great cause of a large mortality in any population, is the hardship which surrounds infancy, among the lower classes.* Now, such hardship must obviously increase, as the circumstances of the poor deteriorate, and *vice versa*. But an undo increase of population is a leading cause of this deterioration; and therefore, an excessive increase in the cipher of births, will generally produce a corresponding increase in the cipher of deaths" (p. 10 my italics).

The general tendency of humans to increase geometrically against limited resources causes hardship and the hardship causes death. The fixed relation between mortality and fecundity is a consequence of the Malthusian law. And, as we have seen, it is this relation that provides the force that Darwin thinks ultimately adapts forms to local environmental conditions. Yet, of course, Anonymous's remarks are contrary to the spirit of Quetelet's statistical techniques.

In the second occasion Anonymous discusses directly Quetelet's views of Malthus's law of population growth:

"On the general question of population, Mons. Quetelet agrees *very closely* with the views of Malthus, which he reduces to the following formula: Population tends to increase in a geometrical ratio. The sum of the obstacles, which are opposed to this tendency is, *ceteris paribus*, as the square of the rapidity of actual

² My analysis of Quetelet is supported in large parts by Fisher (1953) and to a greater extent Yule (1928). I think most later commentators on Quetelet (e.g. Hilts (1973)) completely misses the point about what constitutes a large-scale regularity for Quetelet.

increase—another instance of the analogy often found to subsist between *mechanical laws* and those which govern human action" (p. 13, my italics).

Since Darwin's intent was to determine the God-given laws of nature that dictate the cause of extinctions and the production of new forms, Darwin might have found the law-like formulation of Malthus's theory as an indication that he was on the right track with Malthus. Compare Anonymous's passage with that of Darwin's excited endorsement of Malthus's population law in E5. As Darwin was searching for a fixed law to explain population phenomena like extinctions, he might have found Anonymous's remarks reinforcing.

Anonymous misleads on Quetelet's position on Malthus. As we have seen in our discussion of Quetelet's critique of the "*modus operandi*" of Malthus's population law, Quetelet does *not* agree very closely with Malthus. Anonymous cites Quetelet's passage concerning 'nature levying her tribute of deaths in detail...' (quoted above) then responds: "This is the actual state of most European populations. A large tribute of deaths is taken, by crime and privation, but destructive famines are rare" (p. 13). Anonymous misses the point. The Quetelet quote comes at the end of a discussion of the "*modus operandi*" of Malthus's resolution of the differing growth rates between populations and resources. The point is not that famines are rare but that Malthus's whole vision of the violent crush of populations causing conditions of misery and hardship is misguided. Anonymous fails to discuss the meaning of Quetelet's analogy with clouds or to mention anything about populations approaching their limit as a stable equilibrium. Had Darwin read Anonymous he would not have had a chance to appreciate an early and powerful critique of Malthus's law of populations that could have undermined Darwin's decision to make Malthus the trigger for natural selection.³

So far, we have Darwin utilizing Malthus's growth law to solve the problems of 'population, depopulation, and the formation of new forms'. Anonymous supplies reinforcing comments on the value of Malthus's law instead of Quetelet's critique. But what about those sex ratios? Next, I compare Anonymous's report with what Quetelet actually says. Herschel, in his 1850 review, better than Anonymous, keenly understands the importance of Quetelet's theory of sex ratio as a showcase for Quetelet's statistical technique of causal analysis. I can see why Maxwell and Galton were so impressed.

4. SKEWED SEX RATIOS

Recall that Quetelet's contribution to demography was to borrow techniques from astronomy to demonstrate that large-scale demographic regularities are not, as some would have it, due to God's divine intervention, or fixed laws imposed by God, but emergent from the aggregate of the local events. As Laplace put it: "this regularity is only the development of the respective possibilities of simple events which ought to present themselves more often when they are more probable" (p. 70). Again, recall that the key is what you can derive out of the distribution pattern that emerges from the aggregate of individuals.

³ See Radick (2004) for a discussion of the wider social reasons why Darwin would have been so wedded to Malthus's law.

Quetelet defined his central concept, the average man, around this insight. The average man, as found in Quetelet's more rhetorical descriptions, captured the egalitarian idea of the common man and invoked the promise and hope for the possibility of social reform (Stigler, 1986, p. 171). Anonymous approves of the use of the "average man" as it is useful for physical and medical measurement, and presents a long discussion of Quetelet's methodology of applying statistical tables to test particular hypotheses about the causes of social regularities. But Anonymous disapproves of Quetelet's application of the "average man" to moral measurements. For one, "the manifestations of a moral quality are not purely physical, they cannot be the subjects of measurements" (Anonymous, p. 5). Further, Anonymous takes issue with some of the Quetelet's (excessive) rhetoric of the beauty and goodness of the average man, finding "that the "l'homme moyen" would be a mediocre personage"(p. 21; see also Stigler, p. 171).

Yet, Quetelet went much further than merely gather data to measure various features of the "average man". Quetelet recognized the value of distributing the statistical data of an entire population into groups, to determine (in Quetelet's words) "the proper degree of influence of these causes" on the effect in question. That is, he recognized the value applying Laplace's central limit theorem to rank the influence of the common causes that determine the large-scale distributions. Let me describe two instances where Quetelet showcases his techniques, early precursor's to Fisher's own "analysis of variance" techniques. Quetelet gathered the overall conviction rate for all Parisians who committed a crime and compared it to the conviction rates of various subpopulations: males, females, those accused of a crime against a person, those accused of a crime against property, under 30, over 30, appeared to stand trial, failed to appear to stand trial, unable to read or write, able to read or write imperfectly, able to read and write well, has a superior education (1842, p. 105). By noting the deviation of the conviction rates for each of these subpopulations compared to the overall conviction rates (and assuming that these deviations are not due to chance) Quetelet recognized that some of these factors had more or less influence on overall conviction rates than others. He also concluded that the best position for an accused to be in was a well-educated female over thirty, appearing to stand trial (*Ibid.*).

Quetelet applies similar techniques to the question of the causes of the excess of boys to girls at birth. To determine the degree to which climate might have an affect on the sex ratio he examines data from several states and provinces in Europe and finds no discernable difference. Yet, Quetelet is careful to avoid hasty generalizations from his scant data. He is not ready to conclude that climate has no influence on sex ratios *tout court*. Quetelet reports confounding evidence from the data gathered at the Cape of Good Hope. Among "free births" the sex ratio favors females but the reverse is true among slave births. Quetelet concludes that the data is thus far insufficient to warrant conclusions. Quetelet goes on to consider other factors from relevant demographic data: births in town, births in country, legitimate births, illegitimate births, births "from persons whose occupations tend to develop the physical qualities", "of persons whose occupations tend to weaken these powers", "of persons whose occupations are of a mixed kind", and various categories of births from parents of various age differences. Quetelet concludes

"It results from the examination of the probable causes which may produce the inequality between the births of male and female children...that the most influential, if we may trust to the few documents which science at present possesses, is evidently that which the difference of age of the parents produces: we might even think that the other causes which have been pointed out, are in some manner the effects of it" (1842, p. 13).

This is the conclusion that Darwin recognizes in his notebook entry in 1838.

Anonymous gives a fair description of Quetelet's project on causation. Anonymous introduces Quetelet's "fundamental principle" at the outset, demonstrating that the problem of ascertaining the correct theory of the sun's altitude from a variety of observations, the problem of ascertaining whether a die is fair, and the determination of the laws that regulate the development of man are the same according to an application of the law of large numbers. For example, Anonymous writes:

"In any one fair throw, the action of the last set of causes so far overmasters that of the first, that it is impossible to determine what will be the event, scientifically, and not as a mere accidental guess. It is found, however, by observation that in the long run, the reverse is the fact; that the constant causes predominate over the accidental; and that, by embracing a long series of events, an average result may be obtained, which will very nearly approximate to what from theory should happen were the constant causes alone in operation..."

Then, he or she relates the idea to Quetelet's investigations on the laws of development of man,

"it is abundantly clear that the old methods of seeking to establish the true nature of the abstract man...have proved insufficient... and that, therefore, the method of investigation embraced by the term Statistics, would be worth of all attention...It is no small matter to have ascertained that from amidst the chaos of individual action...there arise as the result of every long series of observations...an order and progression of moral cause and effects sufficiently precise to become the subject of general reasonings..."

Schweber (1977) remarks that passages like these provide a clear articulation of Ernst Mayr's notion of "population thinking".

Had Darwin carefully considered such passages it would support Hodge's contention (1987) that by November 1838 Darwin was incorporating a concept of chance referring to outcomes of long run frequencies. In several places where natural selection is invoked or illustrated with an example, Darwin refers to advantageous individuals "hav[ing] a better chance of surviving" relative to their variants. Hodge interprets such passages (as well as others) as "chance" invoked in the context of the law of large numbers. If so, Anonymous's review could have served as reinforcement to Darwin's understanding of chance and causation just as Herschel's review of Quetelet in 1850 is said to have reinforced the value of a frequentist

notion of chance in the minds of Maxwell and Galton.

Yet, there is more to Quetelet's techniques than to utilize the law of large notions to understand how certain variants can increase in frequency in the long run. There is Quetelet's use of comparing statistical tables to test for variance between average results as an important contribution to the investigation of causes. I discussed how Quetelet employs this technique, above, in the discussion of crimes and sex ratios. Herschel, in his 1850 review, calls the statistical technique that Quetelet pioneered "statistical enumeration" (p. 28).

Herschel's review is couched in an historical account of the development of statistics in science (and extremely useful for students of history in this regard). Herschel views Quetelet as an important contributor to the development of the use of statistics in science in general in addition to (and not merely) an application of statistical method to social studies (e.g. p. 16-18). So while Anonymous seeks to tie together astronomy, games of chance and sociology, Herschel views the techniques of applying a distribution to determine the source of cause as valuable for natural investigations in general:

Nothing can be more irregular and uncertain than the action of the wind on the waters,—yet, in the most violent storms, the *general* surface of the ocean preserves its level. What more fortuitous than the fall of a drop of rain in a shower or the growth of a blade of grass? Yet the soil is uniformly irrigated, and the unbroken sheet of verdure testifies to the resultant equilibrium of that and a thousand other causes of inequality. These things, it will perhaps be said, are the results of Providential arrangement. No doubt they are so; but it is an arrangement working through a complication of secondary causes and contingencies...on which man, if he will philosophise at all, is obliged to do it by reference to the laws of probability" (Herschel, p. 30).

In that light, the most important distinction between Anonymous's and Herschel's review is that for Anonymous, Quetelet's central contribution is the formulation of the concept of the average man and its usefulness in various measurements for societal features and trends. For Herschel, the average man is just one of Quetelet's valuable contributions to statistical science, never mind the rhetoric and morality. Quetelet's other important contribution to statistical science the investigation of the causes that determine the averages, his method of what Herschel calls "statistical enumeration" (p. 28). It is not that Anonymous ignores Quetelet's use of statistical tables to determine causes—he or she does mention it—but the discussion is blended with a discussion of the average man which, Anonymous notes, carries with it significant rhetorical baggage. Herschel, on the other hand, articulates and highlights Quetelet's use of comparing statistical tables to test for variance between average results as an important contribution to the investigation of causes, independent of the concept of the average man (Brush 1965).

For Herschel Quetelet's analysis of the skewed sex ratios showcases Quetelet's technique and Herschel devotes considerable attention to a discussion. Anonymous, as with Herschel, describes the conclusion, that the causes of the skewing has something to do with the physical conditions of the parents

given their relative ages making it more likely (in the long run) that more boys will be born than girls (Anonymous, p. 11, Herschel, p. 34). But, to Herschel the sex ratio study demonstrates a deeper lesson about confirmation of constant causes and their distinction to variable and accidental factors. Anonymous provides no such discussion. Herschel summarizes the section as follows:

All experience tells us, that where efficient *causes* are known, but from the complication of circumstances cannot be followed out into their specific results, we may yet often discern plainly enough their *tendencies*, and that these tendencies *do* result, in the long run, in producing a preponderance of events in their favour. Were it asked, Why do the strong men, in a general scramble, carry off the spoil, and the weak get nothing? The reply would be, that such is not the fact in every instance; that, although we cannot go fully into the dynamics of the matter, we can clearly see the mode of action in some individual struggles, and that in the whole affair there is a visible enough *tendency* to the defeat of the weaker party. Again, when we reverse this process of reasoning, and declare our conviction that success in the long run is a proof of ability, we give this name to some personal quality or assemblage of qualities which, acting as an efficient cause through a complication of events we do not pretend to penetrate, has a tendency in that direction which issues in success...

And thus we are led to perceive the true, and, we may add, the only office of this theory in the research of causes. Properly speaking, it discloses, not causes, but tendencies, working through opportunities... (p. 31-32).

C. C. Gillispie cites this passage and accompanies it with the provocative remark that had Darwin read it he would have had no need to read Malthus (1961, p. 452). I take it that Gillispie, like Quetelet before him, was impressed that there is no need to impose an external force to explain why adaptive individuals win the spoils. In general, adaptive individuals have qualities that enable them to defeat their competitors. While this is not true for any particular individual, nor can we, for any particular individual state exactly what enables them to defeat the weaker, we know the general tendency to be true from taking the long view that large samples of statistical data provide.

5. DARWIN ON QUETELET

While I cannot prove that Darwin ever read Quetelet's work, nor can I prove that Darwin read either the *Atheneum* or *Edinburgh Review* reviews, I can prove that Darwin was familiar with Quetelet's work. Darwin's 1838 notebook entry is not the last time he mentions Quetelet. In 1874, "On the Males and Complemental Males of Certain Cirripedes, and on Rudimentary Structures" Darwin writes:

The following conjectural remarks are made solely in the hope of calling the attention of naturalists to this subject. It is known from the researches of Quetelet on the average height of man, that the number of

individuals who exceed the average height by a given quantity is the same as the number of those who are shorter than the average by the same quantity; so that men may be grouped symmetrically about the average with reference to their height....So it is with the circumference of their chests, and we may presume that this is the usual law of variation in all the parts of every species under ordinary conditions of life. That almost every part of the body is capable of independent variation we have good reason to believe, for it is this which gives rise to the individual differences characteristic of all species.

Darwin credits Quetelet for introducing the idea of the *universality* of the normal curve to characterize variation. So, in Darwin's mind, Quetelet's contribution to natural selection is modest. It concerns the character of quantitative variation, *not* the statistical analysis of how distributions are decomposed to determine their causes (or 'tendencies' on Herschel's view). Darwin fails to describe the central insights that Quetelet offers. This, I think, gives the final vindication to Fisher's remarks about Darwin failing to appreciate what Quetelet has to offer Darwin's theory of natural selection.

The tendency of Quetelet to find normal curves in statistical sample has been dubbed by Edgeworth in 1922 as a "Quetelismus", on the ground that Quetelet exaggerated the prevalence of the normal law (Stigler 1986, p. 203).

CONCLUSION

In 1838, Darwin was interested in solving an instance of the problem of populations, how extinctions arise out of multiple causes. He found in Malthus a way to do it with a single fixed law of nature that impinges its effect on every member of the population, the sum effect is a collective force for which populations crush against the boundary of resource constraints. A warring struggle for resources follows. Any small change of ecological conditions triggers a cascade of evolutionary effects, some forms force out others, new forms populate. While this is not the Darwin we know from the *Origins*, even then, Malthus's law of excessive reproduction acts like an external trigger for evolutionary change. Having seized this solution to Darwin's puzzles about populations he saw no need for Quetelet's statistical techniques. He found a fixed causal law that instigates change and acts like a force. But even if Darwin had needed Quetelet there was not much in Anonymous to convince Darwin that Malthus does not hold the answer or that Quetelet's techniques provides a suitable alternative. Yet, Malthus is a long way from the statistical approach of Quetelet and Fisher. Fisher says this best:

"Both Darwin and Wallace ascribe their first confident appreciation of the potency of Natural Selection, not to personal contact with Malthus, but to the effects of casually reading his Essay on Population. How easy it would be to say, "Here is the source of that subtle statistical argument which convinced both Darwin and Wallace that they had uncovered the effective cause of Organic Evolution". I do not say it, because I do not believe it."

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