Anhui University

Semester 1, 2004-2005 Final Examination (Paper A)

Subject title: ______________________________

Time allowed: 2 hours **School or Department:** ______________
Major: ______________ **Student Name:** ______________
Student Number: ______________ **Seat Number:** ______________

Question No.	1	2	3	4	5	6	7	8	9	10	11	Referee	Score
Marks													

In question 1-6, please choose the correct answer (only one is correct)

1. (5 marks) Word "MATLAB" comes form
(A) Mathematics Laboratory
(B) Matrix Laboratory
(C) Mathematica Laboratory
(D) Maple Laboratory

2. (5 marks) The matrix

$$\begin{pmatrix} 4 & -1 & 1 \\ 4 & -8 & 1 \\ -2 & 1 & 5 \end{pmatrix}$$

is
(A) a strictly diagonally dominant matrix;
(B) not a strictly diagonally dominant matrix;
(C) a singular matrix;
(D) a matrix whose determinant is equal to zero.

3. (5 marks) The computational complexity of Gaussian elimination for solving linear equation systems $AX = B$ (A is $N \times N$ matrix) is

(A) $O(N)$, (B) $O(N^2)$, (C) $O(N^3)$, (D) $O(N^4)$.

4. (5 marks) Assume that $f(x)$ is defined on $[a, b]$, which contains equally spaced nodes $x_k = x_0 + hk$. Additionally, assume that $f''(x)$ is continuous on $[a, b]$. If we use Lagrange interpolation polynomial

$$P_1(x) = \sum_{k=0}^{1} f(x_k) L_{1,k}(x)$$

to approximate $f(x)$, then the error $E_1(x)$ is

(A) $O(1)$ (B) $O(h)$ (C) $O(h^2)$ (D) $O(h^3)$

5. (5 marks) Degree 4 Chebyshev Polynomial $T_4(x)$ is

(A) an odd function, (B) an even function
(C) not odd or even function, (D) both odd and even function

6. (5 marks) Padé Approximation is

(A) A rational polynomial approximation
(B) A polynomial approximation
(C) A triangular polynomial approximation
(D) A linear function approximation

7. (15 marks) Use the false position method to find the root of $x \sin(x) - 1 = 0$ that is located in the interval $[0, 2]$ (the function $\sin(x)$ is evaluated in radians).

8. (15 marks) In the following linear equation systems

$$\begin{aligned} 4x - y + z &= 7 \\ 4x - 8y + z &= -21 \\ -2x + y - 5z &= 15, \end{aligned}$$

start with $\boldsymbol{P}_0 = (1, 2, 2)$, and use Gauss-Seidel iteration to find $\boldsymbol{P}_k$ for $k = 1, 2$. Will Gauss-Seidel iteration converge to the solution?

9. (15 marks) Consider $y = f(x) = \cos(x)$ over $[0.0, 1.2]$. Use the three nodes $x_0 = 0.0, x_1 = 0.6$, and $x_2 = 1.2$ to construct a quadratic interpolation polynomial $P_2(x)$.

10. (15 marks) Consider $f(x) = 2+\sin(2\sqrt{x})$. Use the composite Simpson rule with 11 sample points to compute an approximation to the integral of $f(x)$ taken over $[1, 6]$.

11. (10 marks) Assume that $g \in C[a, b]$. If the range of the mapping $y = g(x)$ satisfies $y \in [a, b]$ for all $x \in [a, b]$, then g has a fixed point in $[a, b]$.